Analysis of Step Discontinuities on Planar Dielectric Waveguide Containing a Gyrotropic Layer

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Abstract —**Nonreciprocal wave propagation characteristics through step discontinuities on planar dielectric waveguides with a gyrotropic layer such as ferrite are presented. In the proposed nonreciprocal structures, the wave propagates from a dielectric waveguide to a ferrite and dielectric waveguide or vice versa, where two structures are connected to create abrupt discontinuities. Nonreciprocal scattering coefficients for TE excitation are calculated at 35 GHz using the well-known mode-matching method.**

I. Introduction

DIELECTRIC waveguide discontinuities find applications such as gratings on dielectric waveguide at millimeter-wave as well as optical wave frequencies. Also, these discontinuities can occur in component interconnections. The characterizations of such discontinuities have been reported by many authors [1]–[4]. However, the application of these structures to nonreciprocal devices has not drawn much attention. In the past, nonreciprocal phase constants were derived from planar dielectric waveguide structures with a ferrite layer [5]. Those structures are ferrite–dielectric, ferrite–dielectric–conductor, and dielectric–ferrite–conductor in nature. Recently, an isolation mechanism based on the nonreciprocal coupling between a dielectric waveguide and a conductor-backed ferrite slab has been proposed [6], [7].

In this paper, we present nonreciprocal wave propagating structures where nonreciprocal layered waveguides are connected to a dielectric waveguide to create abrupt discontinuities. The layered structures with a ferrite layer show different phase constants under the external dc magnetic field, depending on the direction of wave propagation. Therefore, nonreciprocal scattering phenomena can be obtained from the proposed structures by adjusting the structural parameters in such a way that the phase constants of two waveguide modes are very close in one propagating direction while those in the other direction are different.

Though various analysis methods can be employed for these structures, the modified transverse resonance method [5] was used for the eigenvalue analysis, and the mode-matching method was employed to derive the scattering coefficients at the discontinuities.

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II. Analysis

The two abrupt discontinuity structures are shown in Fig. 1. Each structure consists of two different waveguiding structures. Waveguide A is a dielectric waveguide on a conductor. Waveguide B is a dielectric–ferrite waveguide on a conductor, as shown in Fig. 1(a), to create nonreciprocal properties. Similarly, in Fig. 1(b) waveguide B is a ferrite–dielectric waveguide backed by a conductor. The external magnetic field, H_{ext}, in the ferrite layer is applied in the y direction. We assume that there are no structural variations in the y direction and that there are no dielectric and magnetic losses. The upper ground conductor is placed in such a way that it does not affect the phase constants of guided modes of two guides, but it contributes to discretize the radiation mode spectra [1], [3]. Therefore, the analysis becomes much simpler that that of the open structure. Since TM modes do not show the nonreciprocal phase constants under these structures, only TE modes are considered in the following analysis. The existing modal fields consist of E_y, H_x, and H_z components, and the total fields in each waveguide can be expanded in terms of TE eigenmodes. We first derive the eigenvalue equations to obtain the eigenmodes for the waveguiding structures in Fig. 1, and then the mode-matching procedures are used to derive the scattering characteristics.

A. Eigenvalue Analysis

Using the modified transverse resonance method based on the *ABCD* matrix approach, the eigenvalue equations of the waveguides for two structures are derived. The eigenvalue equations for the structure in Fig. 1(a) are given

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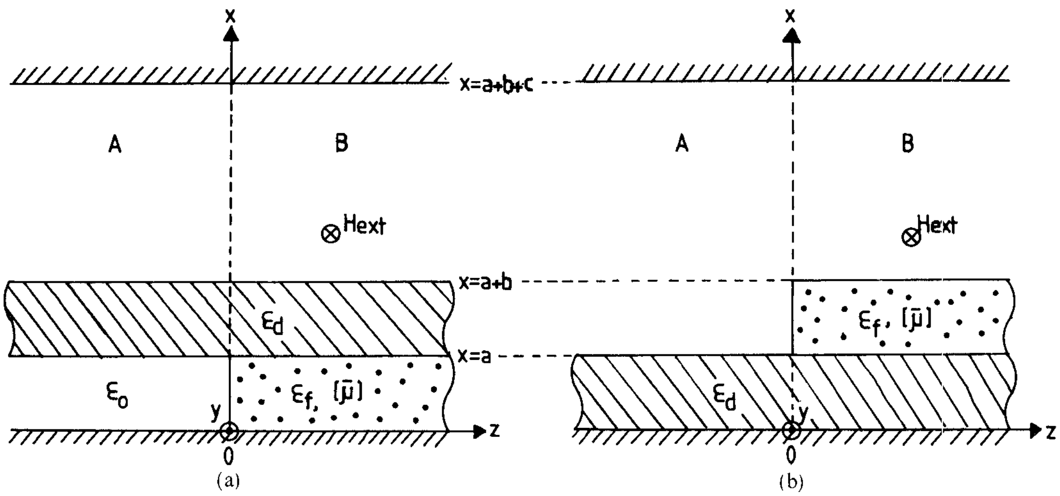


Fig. 1. Step discontinuity structures containing a ferrite. (a) Dielectric waveguide to dielectric–ferrite waveguide. (b) Dielectric waveguide to ferrite–dielectric waveguide.

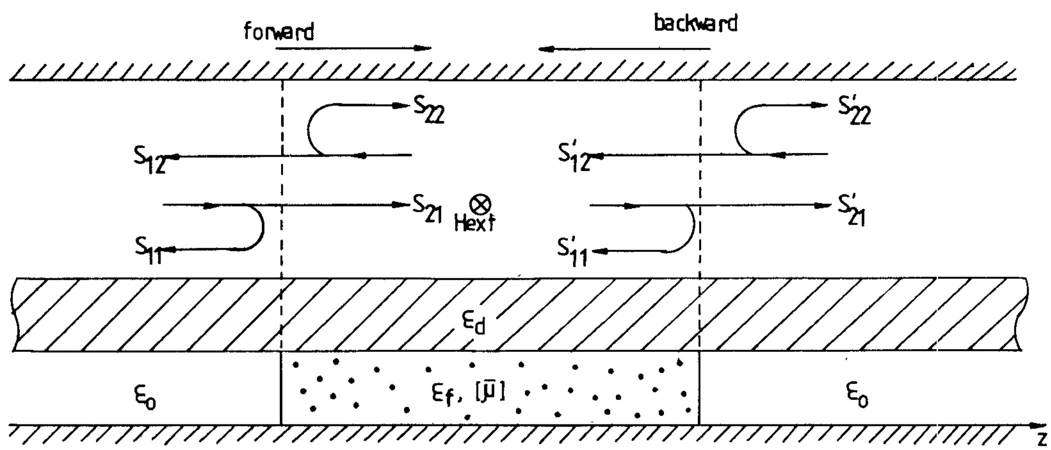


Fig. 2. Convention for scattering parameters for structure in Fig. 1(a).

as

$$1+\frac{k_{dn}^a}{k_{an}^a}\cot\left(k_{dn}^a b\right)\cdot\left[\tan\left(k_{an}^a a\right)+\tan\left(k_{an}^a c\right)\right]-\left(\frac{k_{dn}^a}{k_{an}^a}\right)^2\cdot\tan\left(k_{an}^a c\right)\cdot\tan\left(k_{an}^a a\right)=0 \quad \text{for waveguide A} \quad (1)$$

$$1+\frac{1}{\mu_e}\left[\frac{\tan\left(k_{dn}^b b\right)}{k_{dn}^b}+\frac{\tan\left(k_{an}^b c\right)}{k_{an}^b}\right]\cdot\left[k_{fn}\cot\left(k_{fn}^b a\right)-\sigma\beta_n\right]-\frac{k_{dn}^a}{k_{an}^b}\tan\left(k_{an}^b c\right)\cdot\tan\left(k_{an}^b b\right)=0 \quad \text{for waveguide B} \quad (2)$$

for the structure in Fig. 1(a). Here, k_{dn}^a and k_{an}^a represent the transverse phase constants of the nth mode in the dielectric and the air region, respectively; β_n^a is the phase constant of the nth mode in waveguide A; k_{fn}^b, k_{dn}^b, and k_{an}^b represent the transverse phase constants in the ferrite, dielectric, and air region, respectively; β_n^b is the phase constant of the nth mode in waveguide B; and σ is the ratio of the off-diagonal component κ of the tensor permeability to the diagonal component μ. The eigenvalue equations for the structure in Fig. 1(b) are derived as

$$1+\frac{k_{dn}^{a'}}{k_{an}^{a'}}\cdot\frac{\tan\left[k_{an}^{a'}(b+c)\right]}{\tan\left[k_{dn}^{a'}a\right]}=0 \quad \text{for waveguide A} \quad (3)$$

$$1+\frac{\mu_e k_{an}^{b'}}{k_{fn}^{b'}}\tan\left(k_{fn}^{b'}b\right)\cdot\cot\left(k_{an}^{b'}c\right)-\frac{\sigma\beta_n}{k_{fn}}\tan\left(k_{fn}^{b'}b\right)-\frac{k_{fn}^{b'}}{\mu_e k_{dn}^{b'}}\tan\left(k_{dn}^{b'}a\right)\cdot\tan\left(k_{fn}^{b'}b\right)+\frac{k_{an}^{b'}}{k_{dn}^{b'}}\tan\left(k_{dn}^{b'}a\right)\cdot\cot\left(k_{an}^{b'}c\right)+\frac{\sigma\beta_n}{k_{dn}^{b'}}\frac{k_{an}^{b'}}{k_{fn}^{b'}}\tan\left(k_{dn}^{b'}a\right)\cdot\tan\left(k_{fn}^{b'}b\right)\cdot\cot\left(k_{an}^{b'}c\right)-\frac{\sigma^2\beta_n^2}{\mu_e k_{dn}^{b'}k_{fn}^{b'}}\tan\left(k_{dn}^{b'}a\right)\cdot\tan\left(k_{fn}^{b'}b\right)=0 \quad \text{for waveguide B} \quad (4)$$

where $k_{an}^{a'}$, $k_{dn}^{a'}$, $\beta_n^{a'}$, $k_{fn}^{b'}$, $k_{dn}^{b'}$, $k_{an}^{b'}$, and $\beta_n^{b'}$ have the same meanings as in Fig. 1(a).

By solving the eigenvalue equations (1) through (4), the propagation constants and field components of the bound modes as well as those of the fast modes for the waveguides in Fig. 1 can be obtained. Note that the eigenvalue equations for waveguide B must be solved twice, depending on the direction of either wave propagation or the external magnetic field.

B. Mode-Matching Analysis

The mode matching procedures are now applied at $z = 0$ in Fig. 1 to derive the scattering coefficients. Since the scattering mechanisms at the discontinuities are nonreciprocal, the wave incidence from waveguide A to the discontinuity is considered first. Assuming the lowest bound mode is incident from waveguide A, the scattered fields are composed of the bound as well as fast modes, and the mode-matching equations at the $z = 0$ plane are written as

$$\underbrace{e_1^a + \sum_{n=1}^{\infty} a_n e_n^a}_{\text{waveguide A}} = \underbrace{\sum_{m=1}^{\infty} b_m e_m^b}_{\text{waveguide B}} \tag{5}$$

$$h_1^a - \sum_{n=1}^{\infty} a_n h_n^a = \sum_{m=1}^{\infty} b_m h_m^b \tag{6}$$

where e_n and h_n represent the normalized transverse electric and magnetic field components, respectively, and the superscripts a and b denote waveguides A and B, respectively. The scattering coefficients $\{a_n, b_n\}$ can be obtained approximately by truncating the infinite sums up to N modes and applying the orthogonality relations between the modes. The orthogonality of modes still holds even if there exists a gyrotropic layer [8]. Hence, after imposing the orthogonality conditions given by

$$\int_0^l e_k^a \cdot h_i^a \, dx = \delta_{ki} \tag{7}$$

$$\int_0^l e_k^b \cdot h_i^b \, dx = \delta_{ki}, \qquad l = a + b + c \tag{8}$$

where

$$\delta_{ki} = \begin{cases} 1, & k = i \\ 0, & k \neq i \end{cases}$$

the resulting equations are as follows:

$$\delta_{ik} + a_k = \sum_{m=1}^{\infty} b_m c_{mk} \tag{9}$$

$$\delta_{ik} - a_k = \sum_{m=1}^{\infty} b_m d_{mk} \tag{10}$$

where c_{mk} and d_{mk} are given by

$$c_{mk} = \int_0^l e_m^b \cdot h_k^a \, dx \tag{11}$$

$$d_{mk} = \int_0^l h_m^b \cdot e_k^a \, dx, \qquad m, k = 1, 2, \cdots, N. \tag{12}$$

The mode amplitudes $\{b_n\}$ as well as $\{a_n\}$ are obtained by solving (9) and (10).

Next, we consider the case where the lowest bound mode is incident from waveguide B toward the discontinuity. In this case, the procedures described above can be applied again. The only difference is that the incident wave is the lowest mode in waveguide B. The scattered fields are the same as those given above.

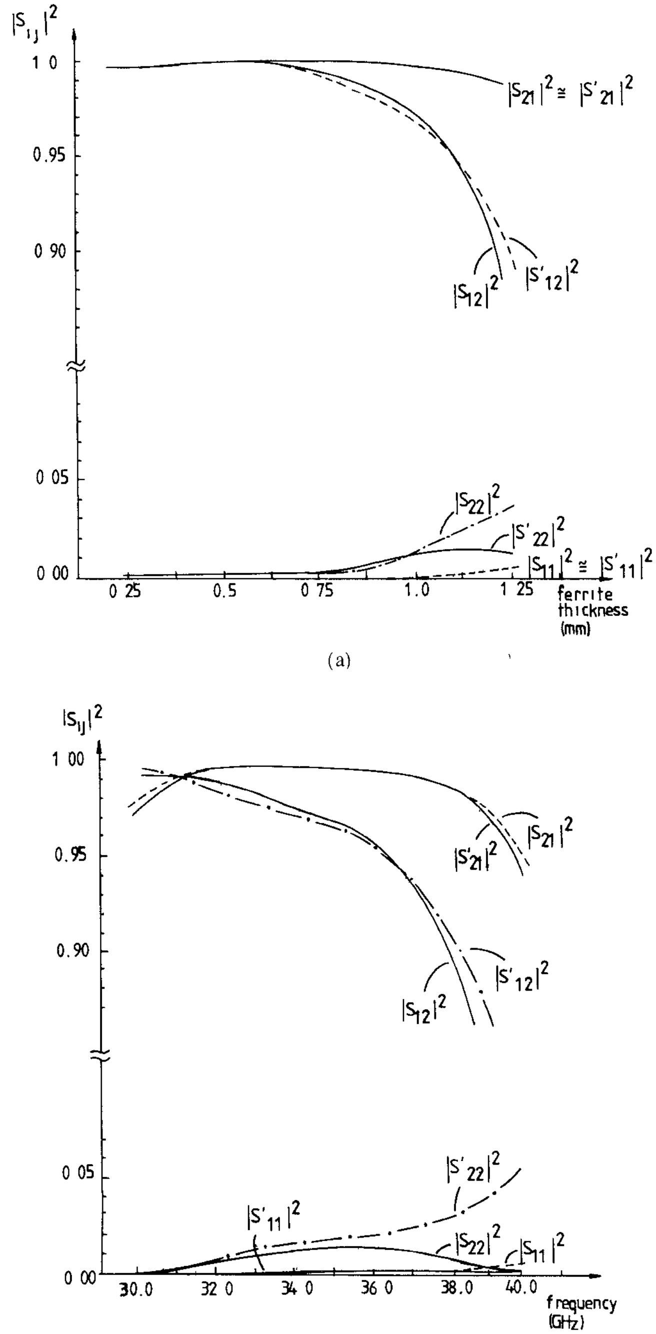


Fig. 3. Scattering parameters for structure in Fig. 1(a). (a) Versus ferrite thickness ($b = 1.0$ mm and $c = 25.0$ mm). (b) Versus frequency ($a = b = 1.0$ mm and $c = 25$ mm).

Finally, we combine the previously derived scattering coefficients to obtain the generalized scattering matrix for each discontinuity structure. When we refer to the scattering parameter convention in Fig. 2, we first derive $[S_{11}]$ and $[S_{21}]$ and, by changing the direction of the external magnetic field, $[S'_{22}]$ and $[S'_{12}]$. Next, we derive $[S_{22}]$, $[S_{12}]$, $[S'_{11}]$, and $[S'_{21}]$ by repeating the above procedures. There-

TABLE I
CALCULATED AMPLITUDE COEFFICIENTS (a_n's AND b_n's) AT 35 GHz FOR THE STRUCTURE IN FIG. 1(a)

n	mode	β_n^f(rad/cm)	$\|S_{11}(n,1)\|^2$	$\|S_{21}(n,1)\|^2$	$\|S'_{11}(n,1)\|^2$	$\|S'_{21}(n,1)\|^2$
1	bound	15.08	0.0015	0.9979	0.0015	0.9981
2	fast	7.23	less than 10^{-4}	less than 10^{-4}	less than 10^{-4}	less than 10^{-4}
3		6.89	less than 10^{-4}	less than 10^{-4}	less than 10^{-4}	less than 10^{-4}
4		6.31	0.0001	less than 10^{-4}	less than 10^{-4}	less than 10^{-4}
5		5.40	0.0003	less than 10^{-4}	less than 10^{-4}	less than 10^{-4}
n	mode	β_n^b(rad/cm)	$\|S_{22}(n,1)\|^2$	$\|S_{12}(n,1)\|^2$	$\|S'_{22}(n,1)\|^2$	$\|S'_{12}(n,1)\|^2$
1	bound	19.84	0.0156	0.9744	0.0180	0.9712
2	fast	7.23	less than 10^{-4}	0.0002	less than 10^{-4}	0.0001
3		6.91	0.0002	0.00010	0.0004	0.0007
4		6.36	0.0007	0.0021	0.0010	0.0018
5		5.49	0.0020	0.0034	0.0021	0.0044

$a = b = 1.0$ mm, $c = 25.0$ mm, and $n = 5$.

fore, we obtain two generalized scattering matrices for each structure in Fig. 1. The characteristics of the scattering matrices derived here will be discussed in detail in the next section.

III. NUMERICAL RESULTS AND DISCUSSIONS

In Fig. 1, we assumed $\epsilon_d = 9.6$ for the dielectric and $\epsilon_f = 11.5$ and $4\pi M_s = 5.0$ kG for the ferrite with external magnetic field strength $H_{\text{ext}} = 5.0$ kOe. The dielectric and magnetic losses are neglected, as assumed previously. The thickness of each waveguide is determined in such a way that the lowest bound mode propagates along the z direction together with the fast modes. The distance between the ground conductors is chosen to be about 27 mm so that not only the phase constant of the bound modes but also the scattering characteristics are unaffected by the upper conductor. The nonreciprocal scattering coefficients of the bound modes for the structure in Fig. 1(a) are calculated following the conventions in Fig. 2. In Fig. 3(a), the calculated scattering coefficients of the bound modes at 35 GHz are plotted as a function of the ferrite thickness, which varies from 0 to 1.25 mm. The thickness of the ferrite is limited to avoid the possible radiation when the structures become open types. Therefore, only the lowest reflected and transmitted modes are considered in Fig. 3 because the magnitudes of the scattering coefficients of the fast modes are negligibly small. However, their magnitudes at 35 GHz are presented in Table I for comparison. In Fig. 3(a), the nonreciprocal scattering coefficients can be observed when the ferrite thickness is greater than 0.7 mm. Therefore, the thickness of the ferrite is taken greater than 0.7 mm in all following discussions. Note also that the scattering matrix for this discontinuity structure has only four components because the magnitudes of the modes, with the exception of the transmitted lowest mode, are negligibly small, as shown in Table I. From the results in Fig. 3(a), we can observe that the magnitudes of the transmission coefficients of the lowest mode propagating in the $+z$ direction (forward direction) are nearly unity, while in the $-z$ direction (backward direction) these magnitudes are much less than unity when the ferrite thickness lies between 0.7 and 1.25 mm. This is mainly because the phase constants β_1^a and β_{1f}^b of the bound modes for waveguides A and B, respectively, are close in the forward direction ($\beta_1^a = 16.48$ rad/cm and $\beta_{1f}^b = 15.08$ rad/cm). However, in the backward direction, the difference between the two phase constants is pronounced ($\beta_1^a = 16.48$ rad/cm and $\beta_{1b}^b = 19.48$ rad/cm).

The frequency characteristics of the scattering coefficients are also calculated and plotted in Fig. 3(b) when the ferrite thickness is chosen as 1.0 mm. We can also observe from Fig. 3(b) that the nonreciprocal scattering characteristics are obtained over a wide frequency range. As the forward transmission characteristics deteriorate, the magnitude of the lowest reflected mode increases. However, the magnitude of the fast modes and the lowest reflected mode remain very small within the frequency range shown in Fig. 3(b). Therefore, this structure is applicable to nonreciprocal devices such as isolators.

A similar numerical analysis is also conducted for the structure in Fig. 1(b). When the ferrite thickness is varied from 0 to 1.25 mm, the nonreciprocal scattering characteristics are also obtained, as shown in Fig. 4(a). The phase constants of the modes in waveguide A are not close to those in waveguide B, i.e., $\beta_1^{a'} = 11.22$ rad/cm, $\beta_{1f}^{b'} = 16.67$ rad/cm, and $\beta_{1b}^{b'} = 19.33$ rad/cm at 35 GHz with $b = 1.0$ mm. Therefore, the magnitude of S_{11} is significant compared with those for the structure in Fig. 1(a). For this reason this structure cannot be applied directly to nonreciprocal devices. But the scattering coefficients of the fast modes are still negligible for this structure. The frequency characteristics of the scattering coefficients for this struc-

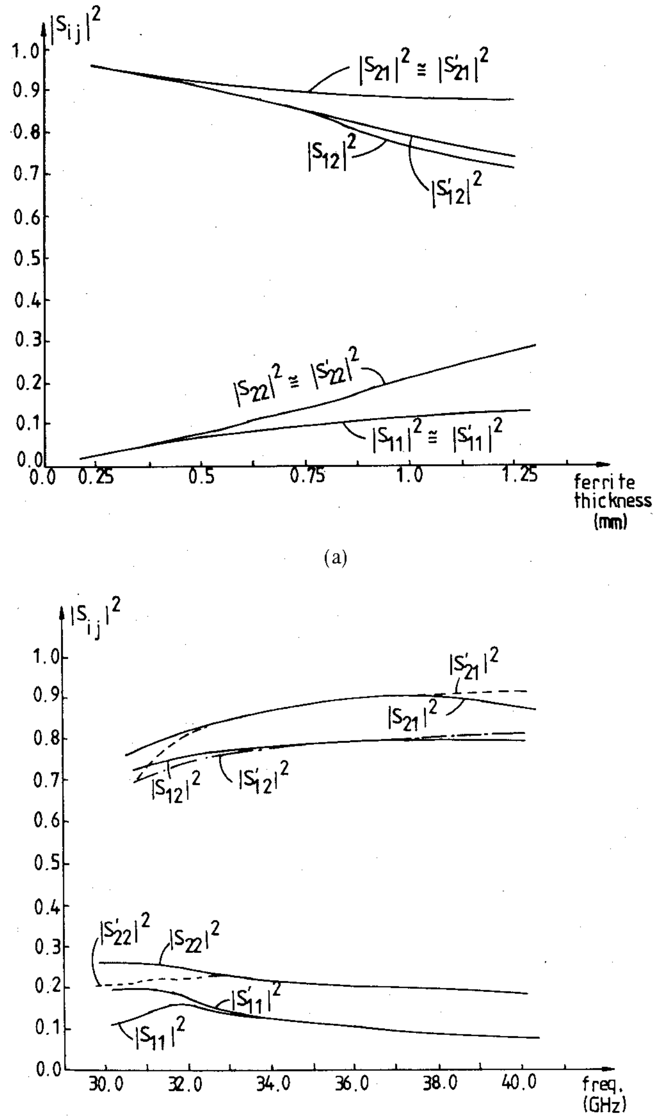


Fig. 4. Scattering parameters for structure in Fig. 1(b). (a) Versus ferrite thickness ($a = 1.0$ mm and $c = 25.0$ mm). (b) Versus frequency ($a = b = 1.0$ mm and $c = 25.0$ mm).

ture, plotted in Fig. 4(b), also reflect the above characteristics.

IV. Conclusions

We have proposed new types of nonreciprocal wave propagating structures composed of a dielectric waveguide and a ferrite and dielectric waveguide. We also presented the calculated scattering characteristics of these structures at 35 GHz. One of the proposed structures is directly applicable to nonreciprocal devices. Nonreciprocal devices made of these discontinuity structures are under study.

References

[1] G. A. Hockham and A. B. Sharpe, "Dielectric-waveguide discontinuities," *Electron. Lett.*, vol. 8, pp. 230–231, May 1972.

[2] T. E. Rozzi, "Rigorous analysis of the step discontinuity in a planar dielectric waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 738–746, Oct. 1978.

[3] G. H. Brooke and M. M. Z. Kharadly, "Scattering by abrupt discontinuities on planar dielectric waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-30, pp. 760–770, May 1982.

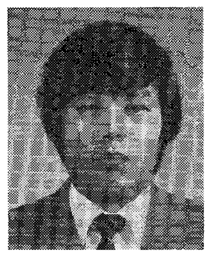
[4] H. Shigesawa and K. Tsuji, "Mode propagation through step discontinuities in dielectric planar waveguide," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 205–211, Feb. 1986.

[5] I. Awai and T. Itoh, "Multilayered open dielectric waveguide with a gyrotropic layer," *Int. J. Infrared and Millimeter Waves*, vol. 2, pp. 1–14, Jan. 1981.

[6] I. Awai and T. Itoh, "Coupled-mode theory analysis of distributed nonreciprocal structures," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-29, pp. 1077–1086, Oct. 1981.

[7] S. W. Yun and T. Itoh, "A novel millimeter-wave isolator," in *Proc. 13th European Microwave Conf.* (Nuremburg), Sept. 1983, pp. 174–178.

[8] D. Marcuse, "Coupled-mode theory for anisotropic optical waveguide," *Bell Syst. Tech. J.*, vol. 54, pp. 985–995, May 1975.



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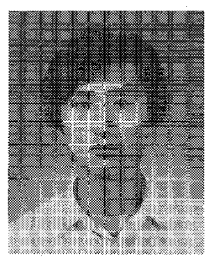

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